Meanings as proposals: an algebraic semantics

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Goal

A semantics that distinguishes:

(1) $p \vee q$

(2) $p \vee q \vee (p \wedge q)$

Approach
(Following Roelofsen 2011)

1. Choose a particular perspective on meaning.
2. Derive a formal semantics from this perspective.

Example: deriving classical semantics

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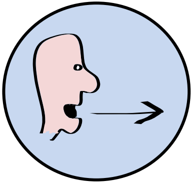
Meaning as information

A meaning A is a set of worlds, $A \subseteq \mathbf{W}$, that represents the information that the actual world lies in A.



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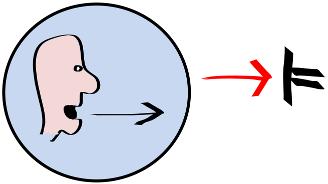

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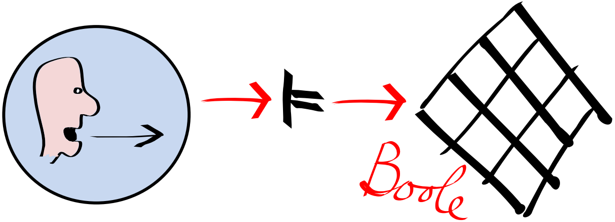

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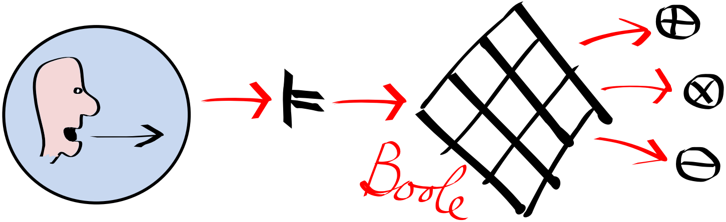

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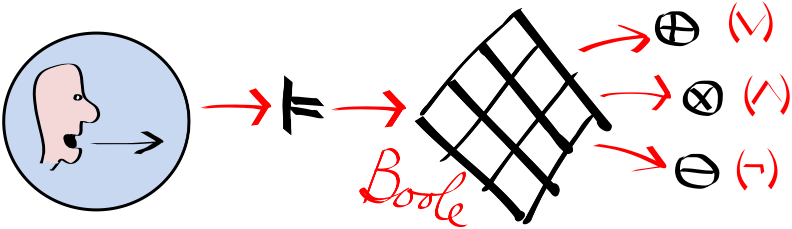

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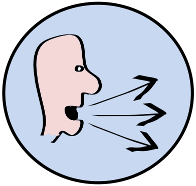

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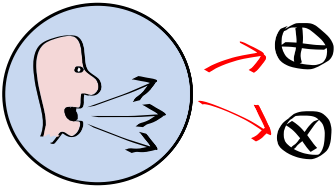

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$A \otimes B := \{f \circ g : f \in A, g \in B\}$

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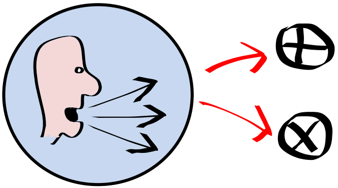

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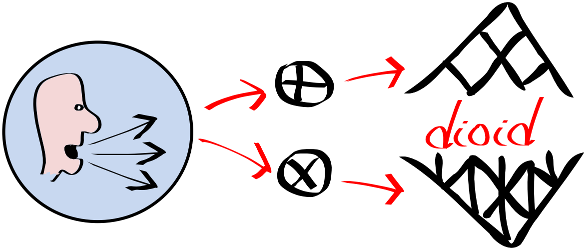

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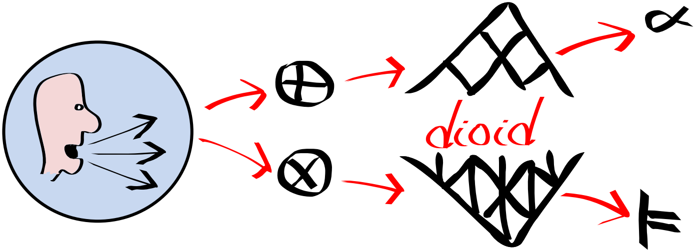

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$\varphi := p|\bot|(\varphi \vee \varphi)|(\varphi \wedge \varphi)|(\varphi \to \psi)$, with $\neg\varphi := \varphi \to \bot$.

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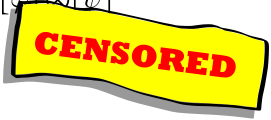
5. $[\varphi \to \psi] =$ 

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For any φ:

- $\bigcup \widehat{[\varphi]} = [\varphi]_{\text{Classical}}$
- $\widehat{[\varphi]}{\downarrow} = [\varphi]_{\text{Inquisitive}}$ (Groenendijk, Roelofsen, et al.)

Fin.

Thanks to the Netherlands Organization for Scientific Research (NWO) for financial support; to F. Roelofsen, J. Groenendijk, J. Marti, I. Ciardelli and an anonymous reviewer for valuable comments.

Miniature posters available!

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